ON THE ECOSYSTEM'S STABILITY

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INTRODUCTION

This report contains an attempt at the theoretical analysis of the types of influence of man on the environment.

Even the simplest mathematical model reveals at least four possibilities — the influence may be pulsed or long-term, it may be exerted on the system itself or affect the regulatory connections.

Therefore, the posing of the question about the maximum permissible loads depends both on the properties of the system and on the nature of the influence.

THE STATE AND THE PROCESS

The state of any system, including an ecological system, is given by the set of numbers characterizing the quantity or level of the components forming this system.

These important variables, which describe the system, are traditionally designated by x with various indexes. The number of variables is determined primarily by the complexity of the system, but depends as well on the desired extent of detailing. Thus, for example, the total number (or biomass) of trees on an area under study may be divided according to species, height, or age.

However, such data are sufficient only for the purposes of classification ("inventory taking"). For the tasks of prediction, and all the more so for the tasks of management, additions and refinements are needed.

The subsequent fate of the system in question essentially depends on the siuation in which it is found — the external environment. The state of the environment in turn is described by some set of numbers. We designate these numbers by the letter y with indexes.

At first glance it appears that we need to bring into the examination "the environment of the environment," to examine another series of letters, then the next one, and so forth, until all existing alphabets have been exhausted.

Strictly speaking, this is true. If, nevertheless, scientific study is at all possible, there must be a serious reason for this. This reason is that each system has its own characteristic time scale and these time scales usually differ radically for the system and the environment containing it.

The stated situation permits a simple and meaningful mathematical formalization:

$$\frac{dx_i}{dt} = f_i(x_1, \ldots, x_n; y_1, \ldots, y_e) \quad 1 \lesseqgtr i \lesseqgtr n$$
$$\frac{dy_k}{dt} = \varepsilon g_k(x_1, \ldots, x_n; y_1, \ldots, y_e) \quad 1 \lesseqgtr k \lesseqgtr e \tag{1}$$

The small parameter ε in this system is equal to the ratio of the characteristic time τ of the system in question to the substantially greater time T, which is necessary for appreciable changes in the properties of the environment:

$$\varepsilon = \frac{\tau}{T} << 1 \tag{2}$$

Thus, for example, the normal frequency of the pulse is one beat per second, and the periodicity of malaria attacks is one day.

Consequently, in this example

$$\varepsilon = \frac{1 \text{ sec}}{1 \text{ day}} = \frac{1}{24 \cdot 60 \cdot 60} \approx 1.1 \times 10^{-5} \tag{3}$$

A medical example and not an ecological one was selected in order to emphasize the common character of the notions being developed.

The structure of system (1) shows that the dynamics, the "life" of the variables x_i occurs against the background of the weak drift of slow evolution of the external parameters y_k. In many instances it is sufficient to set $\varepsilon = 0$.

The variables y_k then become constants:

$$y_k = \alpha_k \tag{4}$$

and enter as parameters the right side of the equations for dynamic variables

$$\frac{dx_i}{dt} = f_i(x_1, \ldots, x_n; \alpha_1, \ldots, \alpha_e). \tag{5}$$

Thus, for example, a sea shell might be found in the zone of alpine meadows at an altitude of 2500

2nd Joint U.S./USSR Symposium on the Comprehensive Analysis of the Environment.
Washington, D.C. : U.S. Environ. Protection Agency, 1975.

meters above sea level. This internally contradictory statement signifies that we are not interested in the geological processes which resulted in the raising of the former ocean bottom to nearly 3 kilometers. We ignore the "geological epsilon."

FIXED REGIMES

The ignoring of the small parameter ε means consequently the fixing of external conditions. However, the state of the system in question may be completely different when given the same values of the external parameters $y = \alpha$.

A forest in a given area may be mature and healthy — this is one fixed state. The overeating of leaves by caterpillars will not kill the forest, but it will lead to another fixed state with sharply reduced photosynthesis. Finally, a fire, having consumed the forest, creates a third fixed state which subsequently will slowly evolve under fixed external conditions.

Mathematically this means that the equation for x may have several fixed states with the given parameters.

The basic ideas can be illustrated by the very simple example of one variable x and one parameter α;

$$\frac{dx}{dt} = f(x, \alpha) \tag{6}$$

In this case the set of fixed states of the system which is given by the equation

$$0 = f(x, \alpha) \tag{7}$$

is mapped by a curve on the plane (x, α).

The situation depicted above causes natural association with the universal biological notion of the states of activity and rest, which are characteristic of all biological systems. There is no doubt that such states are also characteristic for ecological systems. Moreover, the general mathematical approach is also fruitful in the analysis of social, technological and technical systems.

However, it is better to retain the biological, or, even, the strictly medical terminology, owing to the fact that the questions under scrutiny have been studied most of all in medical practice.

PULSED INFLUENCE

The states of activity and rest have a definite stability. The proposed model makes it possible to examine the basic types of reaction of a system to pulsed influences. It is natural to interpret such an influence as an instantaneous transfer from one point of the phase plane to another.

As was already said, visual biological representations on an organism level are the basis. The integration of the notions in the model makes it possible to construct their ecological analogue.

Let us examine the obvious possibility, when the state of rest is sleep, and the state of activity is awakeness. In this case the value x should be interpreted as the level of motor activity, while the parameter α should be tied with the level of excitation of the nervous system.

It is evident that this is an extremely simplified, illustrative description. Nevertheless, it is useful for an understanding of the possibility of a unified mathematical model that does not depend on the structural, morphological level of the system in question.

Thus, for example, it is possible to attempt to analogize the state of "activity" with the golden age of the Helladics, when the entire peninsula was covered with mighty oak forests. At that time the state of "rest" of this ecological system was its present dense condition of thorny bushes and outcrops of rocks. It is believed that the main cause was the goats which had not so much eaten up as they trampled down the underbrush. Freed mountain streams washed away the soil, and karst depressions completed the destruction. And now there are almost no goats and the streams do not rage. . . .

Let us return, however, to the model (Figure 1) and examine the state of activity A, which is on the branch AA′. The pulsed influence on the system corresponds to the instantaneous displacement along the horizontal line which passes through point A. The nonlinear theory of oscillations suggests the name "phase impact" for such a change of state of the system. Such terminology is justified by the extensively widespread name "phase space" for the space of dynamic variables.

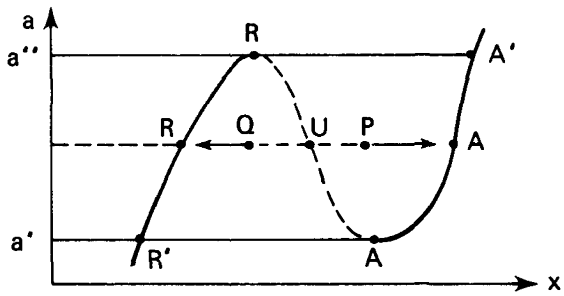

Figure 1. A system with a different number of fixed regimes. In the zone between α' and α'', there are three fixed regimes, whereas above and below this zone there is one for each.

Of course, the phase impact upsets the equilibrium, but if the disturbance has not thrown the representing point of the system beyond the line RA of the unstable states of equilibrium, then the system in agreement with the equation of motion (6) returns to the previous state of activity A. If the phase impact throws the system beyond the point U (on the branch RA), then the system enters a state of equilibrium at point R on the line of the states of rest.

Thus, the phase impact has a clearly defined threshold nature — to the right of U there is the full reestablishment of activity to the original level, to the left of U the system enters the state of rest.

It is necessary, of course, to bear in mind the arbitrary nature of the terminology — the state R should be considered "rest" only with respect to the state A. Thus, for example, a marmot may be awake or be asleep, or may become lethargic. The state of "lethargy" is "rest" with respect to activity, while sleep is rest with respect to lethargy. For our purposes it is sufficient to distinguish between two contiguous levels which differ with sufficient force in the intensity of the activity.

Let us now examine the consequences of the pulsed influence on the parameters of the system. The space of the parameters is called the structural space of the system, since to each point in this space there corresponds a completely defined nature of the dynamics of the system, its very own, as is said in the theory of oscillations, "phase portrait" of the system. Therefore, it is reasonable to call the pulsed influence on the parameters of the system a "structural shift."

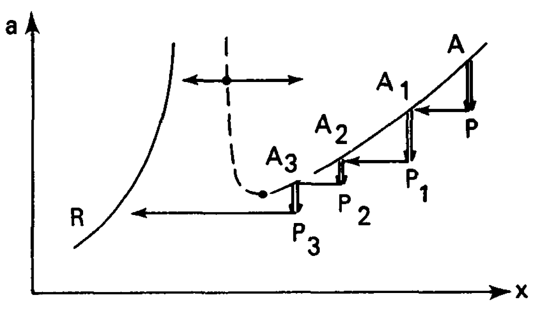

Figure 2. The irreversibility of the structural shift. Following the shift AP, the system enters the balanced working regime with a higher level of activity.

Mathematically a structural shift is a displacement in a plane (x, α) along the vertical passing through point A. Obviously, it is worthwhile emphasizing that the plane (x, α) is the direct product of the phase space (the line x) and the structural space (the line α). In general, this is a space of very great dimensionality, but the necessity of a clear depiction makes it necessary to limit ourselves to a very simple case. Besides, the basic concepts are sufficiently meaningful and rich even given this most simple case.

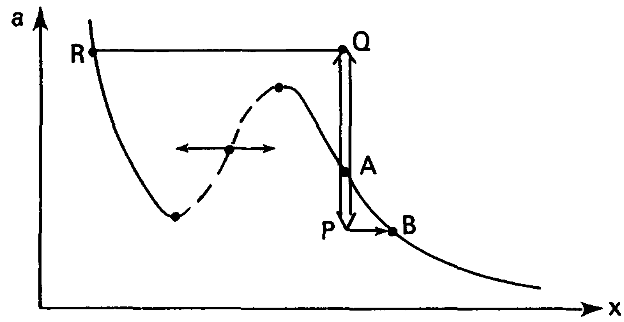

Figure 3. The gradual accumulation of structural reconstructions, which results in a breakdown into the inactive state R.

In contrast to the phase impact, the structural shift necessarily changes the state of the system — there is an "after effect."

The new regime that arises upon the achievement of equilibrium, which was disturbed by the structural shift, may be rest, may be a state of greater or less activity.

Another important property of structural shifts, which is closely connected with irreversibility, is the cumulative nature of such influences.

SLOW (EVOLUTIONARY) MOTION

Everything expounded above pertained only to rapid motions.

The next problem in difficulty is the calculation of slow changes of parameters. For lack of a better word we will call it the "evolution" of the system. However, it is necessary to bear in mind that this is not necessarily evolution in Darwin's sense.

The rapid motion of the variables x we would do well to call the kinetics, the dynamics of the system, while the slow internal structural changes of the parameters would be best characterized by the word "evolution," which opposes them verbally to the kinetics of the system. Thus, for example, the age changes of an ecosystem or organism are naturally called an evolution in respect to vital functions, metabolism and kinetics.

In order to emphasize that we are getting ready to examine an expanded system, let us return to the designation y for slow variables. They have ceased to be external parameters and have become equivalent, even though slow, yet all the same variables of the system.

Here is a simple example. In studying a forest, one might not be interested in the process of soil formation and might consider the soil qualities a given parameter. However, if it is a matter of hundreds and thousands of years, the standing timber takes an active and important part in the creation and change of the soil on which it grows. The reversion to y means, consequently, not only the expansion of the system, but also the significant increase in the time scale during which the study of the system takes place. Great time scales, let us say, geological ones, may no longer be included in such an examination. In conformity with this, the landscape features — river valleys, hills, water-tight layers — also must be considered invariable parameters even for an expanded system.

Let us write out a more complete model:

$$\left.\begin{aligned} \frac{dx}{dt} &= f(x, y) \\ \frac{dy}{dt} &= \varepsilon g(x, y) \end{aligned}\right\} \qquad (8)$$

The points on the curve $f(x, y) = 0$ are no longer stationary points of our complete system.

Nevertheless, the motion in the vicinity of this line occurs considerably slower, with a velocity on the order of ε, and not one, as at the remaining points of the plane (x, y).

The points of the curve $f(x, y) = 0$ are called points of quasi-equilibrium, while those points which "attract" the rapid variables are called metastable. The points of true equilibrium, which correspond to the disappearance of both velocities (both rapid and slow motion)

$$\left.\begin{aligned} f(x, y) &= 0 \\ g(x, y) &= 0 \end{aligned}\right\} \qquad (9)$$

lie, of course, on the curve of quasi-equilibrium, and more precisely, at its intersection with the curve $g(x, y) = 0$.

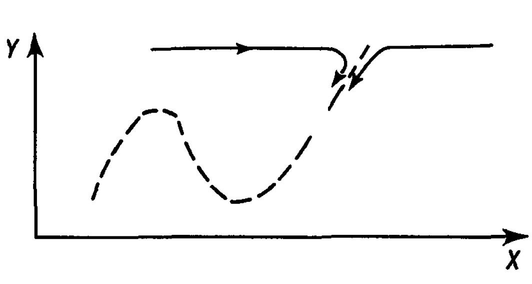

Figure 4. Rapid motion toward the line of quasi-equilibrium f(x,y) = 0, and slow evolution along it.

It stands to reason that this "true" equilibrium can be (and necessarily is) in turn a quasi-equilibrium in respect to even slower motions. We are assuming, of course, that the problem under discussion is correctly stated, for the necessary time scale, with consideration of all significant variables.

Thus, the existence of two time scales leads to two concepts of stability — metastability and complete (true) stability.

It should, perhaps, be noted that the hierarchy in the concept of stability is a reflection and consequence of a profound case — the hierarchy in the structure of the system being studied. Metastability and stability (for the sake of brevity we will not add each time the adjective "true") is the mathematical form of the important features of the structure of complex biological systems.

RAPID AND SLOW MOTIONS

The distribution of the points of equilibrium on the curve of equasi-equilibrium is of decisive significance to the properties of the system and the nature of its reaction to external interference.

Let us examine the case depicted in Figure 5, where the system has a stable equilibrium on the working branch AS, and on the branch of rest the unstable equilibrium U.

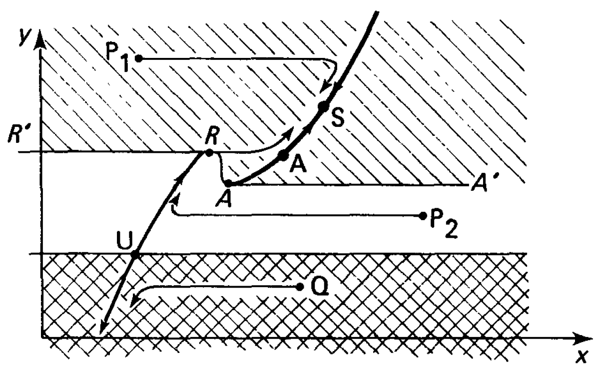

Figure 5. The point S is stable; the point U is unstable. From P_1 and P_2, the system returns to S. From point Q, there is no return.

Assume that the system experienced both a phase impact and a structural shift which threw it to point p_1. Then the system quickly reestablished its working ability, and hence slowly returns to the stable working point S.

The word "quickly" here and henceforth means "after a time on the order of one," and "slowly" — "after a time on the order of $1/\varepsilon$."

The system behaves differently when thrown to point p_2. At first it is even more active (x is greater than S), but this is "unhealthy excitation" and quickly "having expended its forces" the system falls on the branch of rest UR. Afterwards there occurs a slow "reestablishment of forces" — evolution to point R — then a return to a working state at point A. The evolution along the arc AS leads to the complete reestablishment of the original optimum state S.

The entire description is reminiscent of the history of a serious illness with a favorable outcome. For a more substantive understanding of the words "quickly" and "slowly" let us cite an ecological example. In the opinion of specialists, the already mentioned destruction of the forest in Greece occurred over two or three centuries, while for its natural reestablishment (evolution to point R) from ten to one hundred *thousand* years will be required.

Events develop even more dramatically when SQ is disturbed. From point Q the system quickly enters a "shock" state on the branch RU below point U and then there develops "progressive deterioration" — slow evolution draws the system further and further away from point S.

The entire plane (x, y) decomposes in the examined case into three domains.

The domain of stability lies above the line R′RAA′. Between the line R′RAA′ and the horizontal straight line passing through point U there is located *the domain of adaptation.*

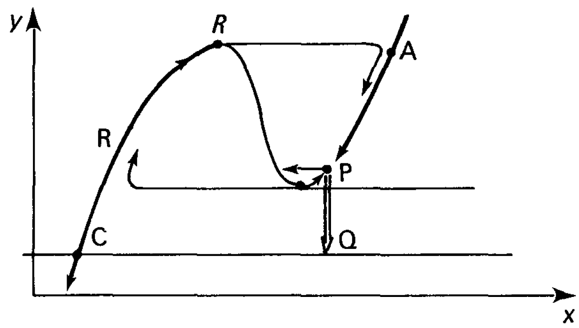

Figure 6. An adaptive system. The domain above the horizontal C is the domain of adaptiveness.

Below the horizontal of U is the domain of depression of the system, if by this we mean the inability to return independently to the state of original activity.

STABLE AND ADAPTIVE SYSTEMS

The existing biological systems have covered a long evolutionary (in Darwin's sense) path. Any of them have both stability and adaptiveness. But different systems have the properties in different proportions. This pertains especially to the ecological systems found under extreme conditions — tundra, desert, mountainous, saline. Unfortunately, this list has now been noticeably expanded by the irresponsibility of mankind.

It is thus more important to examine two extreme cases — adaptive systems with little stability and stable systems with little adaptiveness.

Let us begin with an example of an adaptive system.

The system loses its activity even with weak phase impacts, such as, for example, SU. It is even more sensitive to structural shifts. The shift SP already leads to a quick loss of activity and long recovery period R*R*. However, the system is capable of self-recovery and long-term activity in the sector of evolution AS. Moreover, even comparatively strong shocks such as the great structural shift PQ do not disrupt the system and even do not increase *significantly* the length of the recovery period.

Stable systems react differently to interference. Let us examine in detail the same function f(x, y), but with a different arrangement of the points S and C, which is determined, as we saw, by the properties of slow motion, i.e., of the function g(x, y).

The clearest feature of such systems is that they "do not know how to rest." They are able to quickly restore their activity even when there are strong phase impacts and structural shifts. However, the hitting of the branch of rest results in irreversible, progressive depression. For the system depicted in Figure 7, the domain of adaptation is the narrow zone ending with the arc RU.

As a venture it might be proposed that stability is characteristic of systems under favorable external conditions.

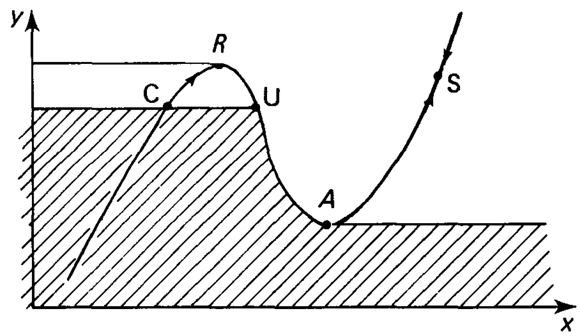

Figure 7. A stable system. The domain of depression begins immediately after the line CUA.

But if the conditions are unfavorable, the system should be adaptive so as not to be destroyed.

METHODOLOGICAL REMARK

From the viewpoint of quick, dynamic phase variables the two examined systems are identical.

The difference between them, and here a fundamental one, is found only with a careful analysis of the evolutionary equation (for slow variables).

Therefore, strictly quantitative approaches (such as, for example, imitation modeling, which was fashionable in the recent past) is suitable for watching after a system, for resolving current, tactical problems.

For the purpose of forecasting, the adoption of long-term solutions, and strategic planning the strictly quantitative methods are entirely insufficient and should be supplemented by a qualitative, systemic, structural analysis of the object in question, by a comprehensive study of the nature of its interaction with the environment and type of reaction to external interference.

HYSTERESIS

In practical work with any complex system — ecological, biological or technical — we usually have no opportunity to "look inside" the system. Therefore interference and direction occur, as a rule, "blindly" — by a change in the parameters of the system and the observation of its reaction.

From this point of view adaptive systems produce a strong impression on the researcher who is accustomed to stable systems. There the situation is simple — to each value of the governing parameter there corresponds a quite definite working regime.

But now the adaptive systems are "capricious." If we give some α today, the system works. If we give the same α tomorrow, the system does not react. And this is in the simplest case, when the system has in all two metastable states.

Meanwhile nothing prevents even a one-dimensional (but complex) system from having several regimes of a differing degree of activity.

In such systems there arise hysteretic phenomena that are described in the simplest case by the concept of the hysteresis loop.

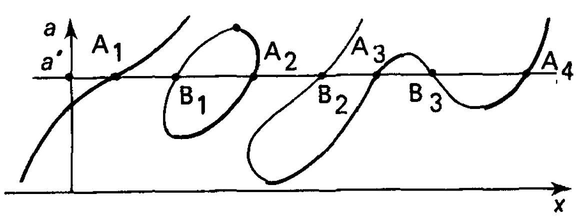

Figure 8. Four metastable regimes, which are divided by three unstable quasi-stationary states.

The phenomena develop in the following way. If the system is initially in state R, then the increase in the governing parameter α beyond the limit α'' leads to a breakdown in regime A. However, the attempt to return to regime R by a rapid decrease in α does not lead to the desired result — the system remains in regime A. There must be a very noticeable decrease in α — below the "lower threshold" of the hysteresis α' — in order to return to the branch of regimes R. In other words, it is possible to approach regime R only from below, and regime A only from above.

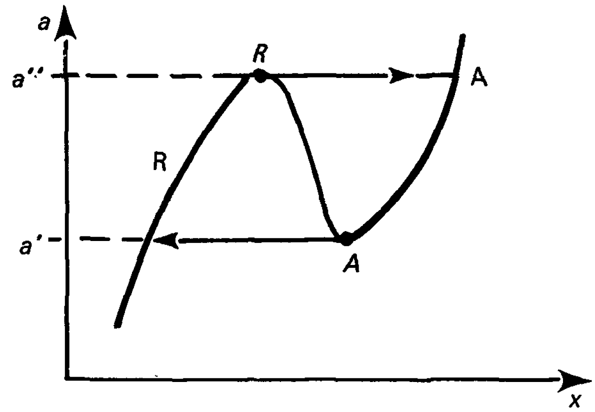

Figure 9. A hysteresis loop formed by the two branches of the regimes A and R.

RELAXATION AUTO-OSCILLATIONS

An additional remarkable situation is a distinctive feature of adaptive systems — they can exist in general without having a stable stationary state. This can easily be seen from the following celebrated example.

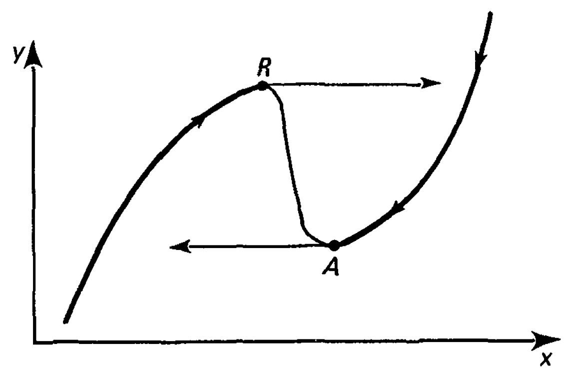

Figure 10. An auto-oscillating regime. A generator of discontinuous oscillations.

In the strict mathematical sense this example was carefully studied in the works of van der Paul, Andronov and others. For our purposes it is important to emphasize that oscillations of this type are not the specific property of radio engineering.

On the contrary, any organism with its clearly periodic alternation of activity and rest is a similar auto-oscillating system. The daily rhythm is a consequence and evolutionary adaptation of an arbitrary, initially auto-oscillating regime.

More complex, ecological systems have adopted (in the middle latitudes) an annual cycle, managing without the external period in the tropics. This attests clearly enough to the endogenic, internal auto-oscillating basis of the adopted (daily, monthly and annual) cycles.

"THE CURSE OF DIMENSIONALITY"

Real biological systems always contain a large number of components of the structural, chemical and morphological type. It seems, therefore, that there must be many variables for the modeling even of not very complex biological systems.

Not by chance do many existing models of ecological systems contain tens and hundreds of variables of the same time scale.

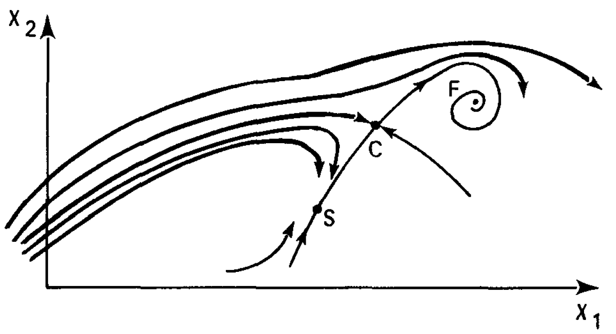

Figure 11. A current pipe. The varying fate of trajectories beginning at near points.

The strictly computational difficulties indeed grow very rapidly as the number of variables grows. This is evident from the following simple discussion. Assume that to study the dynamics of a complex system we calculate on a computer a pencil of trajectories which is "dense" enough so as not to overlook an interesting regime.

Let us assume a net with ten points for each "n" dynamic (phase) variables. Then the total number of trajectories in this current pipe is huge:

$$N = 10^n$$

With the high speed of modern computers of ten billion operations per second ($S = 10^{10}$), in an entire year of continuous calculation (1 year $= 3.15 \times 10^7$ sec) it would be possible to handle a system of the eighteenth order.

A system of the twentieth order would require 100 years.... The fantastic suggestion of increasing the high speed of computers by 10 orders would lead to a system of only the thirtieth order.

All of this means, of course, only one thing: The complete, absolute helplessness of the strictly technical approach, the lack of promise of the methods of direct examination in ecological tasks of even average difficulty.

Only thought, philosophy of life and science can help.

THE BASIC ROLE OF TWO-DIMENSIONAL SYSTEMS

The theory of stability of dynamic systems initially arose in celestial mechanics in the works of Poincaré, Lyapunov and their followers. Subsequent development in the works of Andropov, Chetayev, Bogolyubov, Tikhonov and many others led to the creation of profound qualitative methods of studying general dynamic systems.

For our purposes one simple consequence of the general theory is essential. In order to determine the stability of a stationary state it is necessary to find n characteristic numbers of λ, by solving the age-old equation $\det \| A - \lambda E \| = 0$, where A is the matrix of the linearized system whose coefficients depend, of course, on the parameters of the system.

The characteristic numbers of λ (their n pieces), when n is the dimensionality of the system, which generally speaking are complex, also depend on the parameters. The stability of a stationary state is determined by the signs of the real parts,

$$p = \mathrm{Re}\lambda,$$

of the characteristic numbers.

If all p are negative, $p < 0$, then the stationary state is stable.

However, when the parameters change, the stability may be lost. For this it is sufficient for just one of the p to become zero and then become positive. In all there are just as many numbers as the dimensionality of the system, i.e., there are very many in complex systems. However, "normally" these numbers do not all at once become zero, but only one at a time. Of course, there may be situations in which several p at one time become zero, but for this a very special combination of values of the parameters must be "examined."

This reasoning is not at all strict; it nevertheless shows that more frequent, and thus more important for the applications, is the case when the stability is lost precisely because of one — the only — characteristic number.

This conclusion is very important, for from this it follows that the normal case in the most complex *system is the existence either of two or one significant variable.*

If the real root intersects zero, the one is the significant (unstable) dynamic variable.

But if the complex root becomes purely imaginary, then two significant variables arise.

With subsequent change in the parameters some other pair of variables may lose their stability, but the main occurrences happen precisely with the transition from stability to instability, but not with a complication of the nature of instability.

And it is precisely for these decisive extreme situations that there are serious grounds to doubt that there will be two or even one (in the case of a real root) significant variable.

THE TRANSITIONAL PROCESS OF THE TWO-DIMENSIONAL SYSTEM

What exactly happens after the stability of a stationary point is lost?

In the case of a real root (the unipolar case) there arises the quick motion of the type of the transistion W→A in Figure 1 and the system will simply shift to a new stationary state.

An exactly analogous situation can also arise in the two-dimensional case (the loss of stability of a complex root).

Figure 12 depicts the situation with a "normal" non-extreme value of the parameters of the system. Let the system be in the state S. Let us begin to change the parameters. It may happen that point C will merge with the node S, which will lose its stability and undergo the quick transitional process S→F along the separatrix CF. There will arise a new stationary state — the focus F. It is even easier to imagine the reverse process — the confluence of C with the focus F.

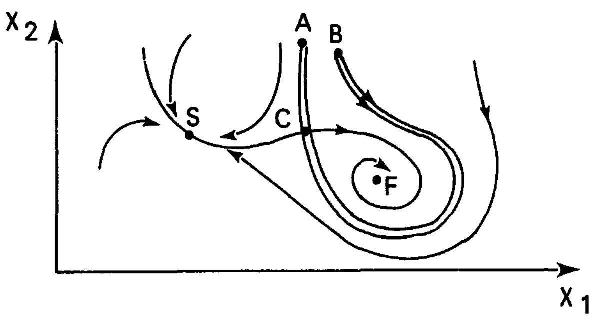

Figure 12. The separatrixes AC and BC isolate the domain of attraction of the focus F. The remaining trajectories bend toward the stable node S.

BIRTH OF THE LIMIT CYCLE

However, in two-dimensional systems there may be a fundamentally new phenomenon — the disappearance of the stationary state and the appearance of a stable periodic regime — the limit cycle.

Let the system whose portrait is depicted in Figure 13 be in the stable state F. The domain of attraction of this state is the interior of the unstable limit cycle C. If when the parameters change the cycle C shrinks into the point F, there occurs *a rigid excitation of oscillations.* The system shifts to an oscillating regime, periodically running over the limit cycle Z.

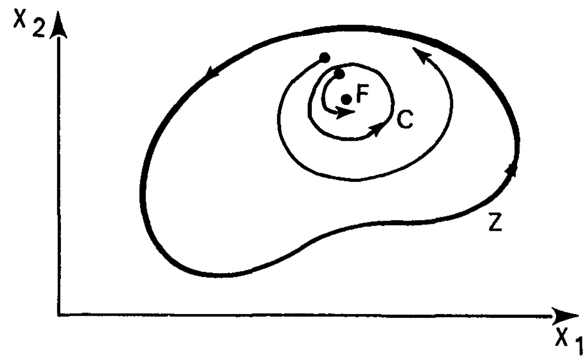

Figure 13. Within the stationary limit cycle is the unstable limit cycle C, which surrounds the stable focus F.

This same effect arises when there is a sufficiently strong phase impact which takes the system beyond the bounds of cycle C. In this instance there also arises a transitional process which does not lead to a new stationary state. As in the first case there arise stable oscillations with a clearly defined period along the stable limit cycle Z.

CONSTANT (FLOW) INFLUENCES

The interaction of man with the environment is not exhausted, of course, by a one-time interference.

More typical is, on the contrary, a constant influence on the system. A typical example is commercial fishing. Annually a certain number of specimens are taken from their populations.

In formal mathematics this is a negative flow in the system. It is influence directly on the system and it can be described by a change in the right side of the equation for x:

$$\frac{dx}{dt} = p + f(x, y).$$

This elementary calculation already shows that the constant influence on the system is more complex than the pulsed influence on the parameters, because it leads not simply to a change in the parameter, but to an increase in the number of parameters, to a change in the dimensionality of the structural space.

The constant influence on the environment corresponds to the appearance of an analogous flow current in the equation for y:

$$\frac{dy}{dt} = \varepsilon\,[q + g(x, y)].$$

An example of this influence is the constant discharge of industrial wastes into a river or lake.

An analysis of possible reactions of systems to such influences which is in any way complete is a complex task.

Even a correct posing of the question offers serious difficulties and should be the object of further research.

It is possible nevertheless not to imagine how such research might develop. The point of departure should be the division of systems into stable and adaptive.

This is evident from the fact that given sufficiently small p and q, adaptive systems remain adaptive, and stable systems, stable.

This simple consideration (the traditional mathematical argument "on continuity") shows the result of research would be, apparently, a more detailed classification of both adaptive and stable systems. Intuitive considerations give grounds to hope that the modern methods of the qualitative theory of ordinary differential equations are quite sufficient for a complete examination of this problem.

The difficulty will most likely be to give a sufficiently rough classification, to avoid the niceties unnecessary for practical work, to which mathematicians are so inclined.

THRESHOLD INFLUENCES

The theoretical analysis made in this report leads to the conclusion:

The traditional differentiation of threshold and cumulative influences on biological (in particular, ecological) systems has reasonable grounds only *under completely defined conditions*:

First, the influence is of an instantaneous, pulsed nature.

Second, the times of observation of the reaction are small in comparison to the time of the spontaneous structural reconstruction of the system.

It also follows from the analysis that a more rough and general description of the properties of the system emerges when introducing the concepts of stability (metastability) and adaptiveness.

These concepts follow from the general concept of stability when considering the hierarchy in the structure of real biological systems, which leads to a hierarchy of radically different time scales.

The question is raised of the more detailed classification of systems according to the reaction to constant (flow) influences.

CONCLUSION

The theoretical study of the problem of stability of ecological systems is a task of great complexity and extreme topicality. It requires the application of an entire arsenal of mathematical means obtained in pre-biological natural science, and, of course, the development of new approaches, ideas and methods.

At present the state of affairs in methodological questions is entirely unsatisfactory.

Even well-known mathematical methods are used in ecological studies with insufficient classification. The well-known methods of Lyapunov are well suited for the description of "dynamic impacts" on the ecological system, of the type of the sudden change in the number of one or several species belonging to the ecosystem. However, the structural shifts that correspond to the parametric influence on the system (the change in the water or salt regime, pollution, etc.) do not have in ecological works any adequate mathematical description.

A disturbing break between theory and practice has arisen and threatens to become entrenched. For questions of long-term forecasting, planning and decision-making it is absolutely necessary to know, what happens when there are structural reconstructions in biosystems? Yet theoretical works repeat in quasi-biological terms the well-known mathematical results, and quite frequently with mistakes.

Meanwhile, quite similar problems have been dealt with for a long time and quite fruitfully in other fields of biology — physiology and biochemistry. In a completely different field of knowledge, engineering, also very great is the role of structural reconstructions, a system having a very specific form of the theory of optimum regulation. In the listed areas, independent contacts have long been developing with mathematics, and definite successes have been achieved.

Consequently, a bountiful collection of specific tasks has been accumulated from a broad circle of branches of knowledge, a collection having nevertheless a profound internal common character. The consistent conducting of mathematical research in this area may lead to the development of a sufficiently general approach — a theory of adaptive systems. The deep internal cause for the possibility of such formalization is the morphological heirarchy of complex biological systems, which is dynamically manifested in the kinetic hierarchy, in the set of motions with a radically different time scale. These properties are manifested most vividly precisely at the organism level, being consolidated by billions of years of biological evolution.

It is useful, therefore, even terminologically ("adaptiveness") to emphasize the desire to incorporate "the lessons of history," the desire to carry over to technological and ecological systems the principles of regulation and management, which have demonstrated their effectiveness in rigid tests of natural selection.

Regardless of the possibility or impossibility of constructing a sufficiently general and meaningful mathematical model, the analogy with a whole organism is useful in itself. This analogy puts in sharp relief the question of creating an adequate system of monitoring. Biology leaves no room for doubt about the importance of the nervous system. Without a nervous system (the internal system of "observation and reporting") there could not be either effective management or even the very existence of any complex system.

Another aspect of this analogy is the selection of subsequent variables that correspond to the dynamic hierarchy of the system. There, also, the role of the study of experimental regimes becomes more comprehensible for revealing the hierarchical structure and construction of an adequate system of monitoring on the basis of indicator types, components, and properties.

Such in general outlines are some of the methodological questions raised before mathematicians by the present state of the problems of environmental protection.